Thermally activated dynamics of the tilts of the CuO$_6$ octahedra, hopping of interstitial O, and possible instability towards the LTT phase in La$_2$CuO$_{4+\delta}$

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I. INTRODUCTION

The oxides of the La$_{2-x}M_x$CuO$_{4+\delta}$ series ($M=$ Sr, Ba) display a complex structural and magnetic phase diagram, and several interconnected phenomena, like lattice instabilities and modulations, charge modulation and segregation, and suppression of the superconductivity at certain values of doping.

The structural phases of La$_{2-x}M_x$CuO$_{4+\delta}$ can be described in terms of tilts of the CuO$_6$ octahedra which compose the lattice around axes which lay in the CuO$_2$ planes, since the CuO$_6$ octahedra are relatively rigid and hinged to each other at their corners, the tilt of one of them propagates to the surrounding ones, resulting in a long-range-ordered pattern of tilts. It is usual to express the rotations in terms of the angles $Q_1$ and $Q_2$ around the [1 1 0] and [1 1 0] axes (the [100] direction in the tetragonal structure is parallel to the in-plane Cu-O bonds). The possible phases are the high-temperature tetragonal (HTT) with $Q_1=Q_2=0$, the low-temperature orthorhombic (LTO) with $|Q_1| \neq |Q_2|=0$ or $|Q_1|=|Q_2| \neq 0$, the low-temperature tetragonal (LTT) with $|Q_1|=|Q_2| \neq 0$, equivalent to a rotation about the [100] axis, and a phase intermediate between the LTO and LTT, generally indexed as Pccn, with $|Q_1| \neq |Q_2| \neq 0$. The HTT phase transforms into the LTO at a temperature $T_{d1}$, which is 530 K for stoichiometric La$_2$CuO$_4$ but strongly decreases on doping by substituting La$^{3+}$ with $M^{2+}$ (Refs. 1, 3) or introducing interstitial O. It should be noted that in the new phase the octahedra are also distorted. The LTO phase tends to transform into LTT or Pccn at a lower temperature $T_{d2}$ in La$_{2-x}M_x$CuO$_{4+\delta}$ ($M=$ Ba, Sr), but the formation of stable LTT domains seems to require a mismatch between the ionic radii in the La sublattice. In fact, when La is partially substituted with the much larger Ba ion, the LTT phase is stable below $T_{d2}\approx 70$ K; instead with Sr, having an intermediate size, the formation of LTT domains in La$_{2-x}$Sr$_x$CuO$_4$ can only be stimulated by the simultaneous partial substitution of La with a smaller ion, like Nd. This is explained by supposing that the distortion of the four octahedra around a pair of Sr$^{2+}$ and Nd$^{3+}$ ions (along c) induces a tilt pattern which is equivalent to the superposition of LTT and LTO domains. There is also a strong interplay between the rotations and distortions of the CuO$_6$ octahedra.
and the electronic properties. Static magnetic and charge modulations have been observed by neutron diffraction to appear in the LTT phase,5,9 favored by the lattice potential of that structure; conversely, it has been proposed that the LTT phase is stabilized around \( x = 1/8 \) by a spin- or charge-density wave commensurate with the lattice and due to charge transfer between the in-plane O atoms which are made inequivalent by the new tilt pattern2,10 (in the LTO phase all the in-plane oxygens are either above or below the plane, while in the LTT phase there are also the O atoms laying on the rotation axes exactly within the plane). The charge-density wave is thought to be the cause of the suppression of superconductivity in a narrow range around \( x = 1/8 \).11,13 The LTT phase is also characterized by structural disorder, and seems to be always accompanied by the intermediate \( Pccn \) one.11,10 Both types of tilts have also been observed in the twin boundary regions of LTO \( \text{La}_{2-x}\text{Ba}_x\text{CuO}_4 \) (Ref. 3), and it is under debate whether the formation of the LTT phase occurs through a widening of the domain walls12 or is mainly nucleated within the LTO domains.13

Signs of the tendency of the LTO lattice to form LTT domains also in \( \text{La}_{2-x}\text{Sr}_x\text{CuO}_4 \) and even in \( \text{La}_2\text{CuO}_4 \) are the large anisotropic vibration amplitudes of the O atoms deduced by neutron diffraction,6 the low energies of the phonon modes involved in the rotations of the octahedra14 and a softening of the in-plane shear below 50 K in \( \text{La}_{1.86}\text{Sr}_{0.14}\text{CuO}_4 \).6,16–19 In addition, there are indications that the octahedra can tilt in a variety of patterns which are different from the average structure observed by diffraction experiments; in particular, the tilts of the octahedra have been observed to persist in the HTT phase,16,19 presumably with very low correlation lengths. Indeed, the instability of the \( \text{MO}_6 \) octahedra against tilting and their tendency to dynamically distort, also forming so-called Jahn-Teller polaron, are features common to several perovskites, including manganites20–22 and the superconductor \( \text{Ba}_{1-x}\text{K}_x\text{BiO}_3 \) (Ref. 23).

In addition, indications exist of interplay between the ordering of interstitial O and the tilt angles.24 In fact, the interstitial O atoms are mobile down to 150 K (Refs. 4,25) and separate into O-rich and O-poor phases; superstructures have also been observed in the O-rich phase due to the segregation of the interstitial atoms in occupied layers spaced by \( n \) empty layers (staging).26,27 and the relationship between the oxygen phase separation and the hole phase separation has to be clarified.28 It appears that the modulations of the octahedral tilts, the ordering and aggregation of interstitial O, the electronic and magnetic phase separations and the superconducting properties are all interconnected to each other.

In spite of the intense experimental activity on these subjects, no indication exists on the dynamics of the octahedral tilts. Regarding the O mobility, the estimates are of indirect type,29 with the exception of preliminary NQR results.29 All these phenomena are coupled to strain, and therefore their dynamical aspects can be studied by acoustic techniques. In fact, the elastic energy absorption and elastic constants, i.e., the imaginary and real part of the strain susceptibilities, are directly connected with the dynamic response of the system, and provide information on phase transformations and relaxation processes. The acoustic investigations on 214 oxides (for a review see Ref. 30) have been so far concerned with the influence of the phase transformations on the elastic moduli;5,15,30–35 only a few deal with the elastic energy dissipation31,32,36 of \( \text{La}_{2-x}\text{Sr}_x\text{CuO}_4 + \delta \) and \( \text{La}_{2-x}\text{Ba}_x\text{CuO}_4 + \delta \), and none with that of \( \text{La}_2\text{CuO}_4 + \delta \). The main feature from ultrasonic and lower frequency measurements is a huge softening of \( c_66 \) connected with the HTT to LTO transformation,5,15,31–33 accompanied by high values of the acoustic absorption from the movement of the twin walls.32 The occurrence of the LTO-LTT transformation has been seen as a much smaller effect both in the sound velocity and absorption of \( \text{La}_{2-x}\text{Ba}_x\text{CuO}_4 \) at \( T_d = 50–65 \) K,34 and as a dip in \( c_66 \) in \( \text{La}_{2-x}\text{Nd}_{x}\text{Sr}_x\text{CuO}_4 \) (Ref. 35), but it has been suggested also for \( \text{La}_{2-x}\text{Sr}_x\text{CuO}_4 \) with \( x \sim 0.1 \), where similar features were found around 10 K in the superconducting state.34

We present the anelastic spectra of \( \text{La}_2\text{CuO}_4 + \delta \) with \( 0 < \delta < 0.02 \) between 1 and 700 K, with four main thermally activated relaxation processes not reported before. Two of them are due to the hopping of excess O in the form of single interstitial ions and pairs or clusters. When the material is outgassed from excess O, the elastic susceptibility of \( \text{La}_2\text{CuO}_4 \) is not as flat as expected from a stoichiometric and defect-free crystal, but is dominated by an intense relaxation process with an activation energy of 0.23 eV, which we identify as thermally activated flipping of a fraction of the \( \text{CuO}_6 \) octahedra between nearly equivalent configurations, presumably within the twin boundaries. In addition, below 30 K still another intense relaxation process occurs, which we attribute to tunneling of lattice O atoms between metastable positions, possibly resulting from the tendency of the LTO lattice to form LTT domains. The dependence of these relaxation processes on doping and lattice disorder is also discussed.

II. EXPERIMENT

The samples of \( \text{La}_2\text{CuO}_4 \) and \( \text{Nd}_2\text{CuO}_4 \) were prepared from powders first treated for 18 h in air at 1050°C (90 h for \( \text{Nd}_2\text{CuO}_4 \), in order to have a single phase), checked by x-ray diffraction, pressed and finally sintered for 18 h in air at 1050 °C. The ingots were cut as bars approximately \( 40 \times 4 \times 0.4 \) mm \(^3\). In order to measure the real and imaginary parts of the dynamic Young’s modulus, they were suspended with thin thermocouple wires and electrostatically excited on their flexural modes; the vibration amplitude was detected by a frequency modulation technique. Since the resonant frequencies \( f \) of the flexural modes are proportional to the square root of the Young’s modulus \( Y \), the dispersion of the real part of the elastic susceptibility was obtained from

\[\frac{dY}{f_0} = \frac{Y}{f_0} \]  

where \( f_0 \) is a reference frequency. The imaginary part is equal to the elastic energy-loss coefficient, \( Q^{-1} \), (reciprocal of the mechanical \( Q \) of the sample) and was measured from the width of the sample resonance or from the rate of the free decay of the vibration. All the measurements were made on cooling at rates below 0.7 K/min.

The contribution of a relaxation process to the dynamic elastic modulus, the Young’s modulus \( Y \) in the present case, is

\[Y = \frac{dY}{df}\]
The O stoichiometry of La$_2$CuO$_4$ was varied by two methods: progressive outgassing in vacuum (sample 3) and thermal treatments in static O$_2$ atmosphere in a quartz tube (sample 2). In the second case, each oxygenation was preceded by an outgassing at 700–750 °C, following the O evolution with a residual gas analyzer, in order to obtain an initial reference state. The sample was then equilibrated at 620 °C for 90–120 min with a static atmosphere of pure O$_2$ and rapidly cooled. The initial pressures of the treatments reported here were 0.4, 0.7, 1, 10, and 835 torr O$_2$; in order to maximize the O uptake at the highest O stoichiometry, after equilibration at 820–835 torr, the sample was cooled slowly at −0.2 °C/min. In this condition the O stoichiometry was estimated to be δ~0.018 from the temperature of the HTT-LTO transformation (see below).

We mention that the values of δ estimated from the amount of gas absorbed were several atomic percent, i.e., higher than the values generally accepted for oxygenation at these pressures. Discrepancies are often reported in literature between the values of δ deduced from the amount of gas absorbed and those deduced from the amount of holes doped in the CuO$_2$ planes (or from the mean valence of Cu), under the hypothesis that each O dopes two holes. The reason for that may be the formation of O complexes which are partially covalently bonded, and therefore introduce less than two holes per O atom.$^{24,38}$

It was also verified that the vacuum treatments did not cause any deterioration of the sample, by reproducing the same anelastic relaxation spectrum of sample 2 at the maximum O stoichiometry after nine outgassing and oxygenation cycles.

III. RESULTS AND DISCUSSION

A. Interstitial oxygen

Figure 2 shows the elastic energy loss of sample 3 between 100 and 400 K, after three outgassing treatments in vacuum up to progressively higher temperatures, as indicated in the figure. Three main maxima appear at 150, 230, and 305 K, labeled as T, O1, and O2, respectively. All of them shift to higher temperature at the higher vibration frequencies, denoting that they are due to thermally activated processes; this is shown in Fig. 3 for the case of intermediate δ. Peak T, which actually consists of two components, is fully developed when the sample is completely outgassed from excess O. The introduction of interstitial O depresses...
The values of the parameters are reported in the text. Finally, becomes predominant. The same phenomenology is observed in sample 2 (Fig. 4), whose stoichiometry was varied by equilibration with different amounts of $O_2$, as explained above. The $Q^{-1}(T)$ curves of sample 2 in Fig. 4 extend the observations to higher values of $\delta$, at which peak $O_2$ reaches a very high intensity. The small peak at 112 K visible at intermediate $O$ concentrations is distinct from peak $T$, since its temperature is lower and does not increase with the $O$ concentration. This peak is also present in sample 3, but at lower temperature because of the lower frequency.

The two processes $O_1$ and $O_2$ are evidently due to interstitial $O$. Since $O_1$ develops at lower values of $\delta$, it must be due to the hopping of single interstitial atoms. It should be noted that the interstitial sites for $O$ described in Refs. 39 and 40 would all be equivalent in a perfectly tetragonal lattice with untitled octahedra; therefore, the hopping of single $O$ atoms among these sites would not change the local distortion and would not cause any anelastic effect [$\lambda_1, \lambda_2$ in Eq. (1) below]. However, since the actual lattice configuration is not tetragonal, some variation of the local elastic distortion is expected when $O$ jumps between neighboring sites. The solid lines in Fig. 3 are a fit with three peaks of the anelastic relaxation spectrum of sample 3 at intermediate concentrations, measured at the three vibration frequencies; each component is the imaginary part of the contribution of a relaxation process to the dynamic elastic modulus [Eq. (1)].

The parameters describing peak $O_1$ in Fig. 3 are $t_0 = (2.5 \pm 0.5) \times 10^{-14}$ s, $E/k_B = (5600 \pm 120)$ K, $\alpha = 0.7$; these values support the hypothesis of $O$ hopping. The value of $t_0$ is typical of atomic hopping with an attempt frequency close to that of the local phonon mode. Furthermore, the hopping rate $\tau^{-1}$ we estimate in excellent agreement with the observation that the mobility of the excess $O$ is frozen below 150–150 K. In fact, this is the temperature range in which the mean time $\tau$ between subsequent jumps passes from 5 min to more than 1 h. The activation enthalpy of $O_1$ is also close to that reported for a relaxation process previously observed by NMR and attributed to $O$ diffusion. On the other hand, the width of the peak is considerably higher than expected from the hopping of an isolated interstitial atom in a regular lattice, and can be explained by the variety of the arrangements of the octahedral CuO$_6$ units, which would give rise to a distribution of local strains. The rearrangements of the lattice structural units may also mediate the interaction between different interstitials; an indication of a correlated dynamics of the $O$ interstitials comes from the broadening of peak $O_1$ at increasing $O$ concentrations, and from its shift to higher temperature, which means a slowing of the relaxation dynamics.

Peak $O_2$ is also certainly connected with excess $O$, both because of its dependence on $\delta$, and because the enormous intensity it can reach is more easily explained by hopping of atoms rather than small atomic rearrangements or polaronic distortions (which generally have smaller $\Delta \lambda$). The most simple mechanism that can be proposed for $O_2$ is the formation of relatively stable pairs of interstitial $O$ atoms, whose local distortion is more anisotropic than that due to the two isolated atoms, and therefore whose reorientation produces intense anelastic relaxation. The reorientation of the pair would require its dissociation and therefore would be slower than the hopping of single interstitials, so justifying the higher temperature of $O_2$ with respect to $O_1$. Such pairs could also be identified with the interstitial $O$ atoms with partially covalent bond, which have been invoked to explain a doping efficiency of less than two holes per excess $O$ atom.

In the doped samples we did not observe any clear sign of phase separation into $O$-rich and $O$-poor phases, nor of
staging\textsuperscript{26,27} within the O-rich phase, but it is possible that these transitions occurred and were masked by the intense relaxations O1 and O2.

**B. Stoichiometric La\textsubscript{2}CuO\textsubscript{4} and comparison with Nd\textsubscript{2}CuO\textsubscript{4}**

Excess oxygen is the main defect of La\textsubscript{2}CuO\textsubscript{4} and after its removal one expects a flattening of the elastic susceptibility below room temperature; on the contrary, we observe the appearance of the intense relaxation process T and the enhancement of another process below 40 K. Figure 5 compares the elastic susceptibility of stoichiometric La\textsubscript{2}CuO\textsubscript{4} (after O outgassing in vacuum at 750 °C for 75 min) with that of Nd\textsubscript{2}CuO\textsubscript{4} (also after annealing in vacuum up to 600 °C, which did not change much the anelastic spectrum compared to the as prepared condition). Both samples were prepared under similar conditions, with comparable dimensions and grain sizes, but the elastic susceptibility of Nd\textsubscript{2}CuO\textsubscript{4} is practically flat at all temperatures, in sharp contrast with that of La\textsubscript{2}CuO\textsubscript{4}. This fact evidently reflects the differences between the two structures: in Nd\textsubscript{2}CuO\textsubscript{4} the O atoms are all stacked along the same lines in the c direction (T’ phase),\textsuperscript{42} so that there are no apical oxygens and CuO\textsubscript{6} octahedral units, but only flat CuO\textsubscript{2} planes. The featureless elastic susceptibility of Nd\textsubscript{2}CuO\textsubscript{4} reflects the absence of any off-center atomic position, lattice instability, or structural phase transformation,\textsuperscript{43} while the changes in the slope of the $Q^{-1}(T)$ (and in the modulus at 20 K, not visible in the scale of Fig. 5) presumably reflect the changes in magnetic ordering observed at 250, 30, and 20 K by neutron-scattering experiments.\textsuperscript{43} The flat anelastic spectrum of Nd\textsubscript{2}CuO\textsubscript{4} confirms the reliability of the measurements and the good quality of the samples, in particular the absence of contributions from spurious phases at the grain boundaries.

The anelastic spectrum of stoichiometric La\textsubscript{2}CuO\textsubscript{4} is dominated by two processes, labeled as T around 150 K and L below 30 K. We will not consider here the minor peaks between 50 and 100 K. Acoustic losses at liquid-helium temperature are generally observed in amorphous materials containing atomic tunnel systems\textsuperscript{44} and even in polycrystalline metals which are not thoroughly purified and annealed.\textsuperscript{45} Such losses however never exceed $Q^{-1} \approx 10^{-3}$, even in fully amorphous samples, and therefore we exclude that peak L, whose intensity reaches $Q^{-1} \approx 10^{-2}$, is due to tunnel systems in spurious amorphous phases.

All the features of the susceptibility curves of La\textsubscript{2}CuO\textsubscript{4} in Fig. 5 are shifted to higher temperature at the higher vibration frequency, indicating that they are all due to anelastic relaxation processes describable by Eq. (1), although the temperature shift of L is much less pronounced, as discussed later. The amplitudes of the steps in the modulus of both L and T are higher than twice the amplitudes of the imaginary peaks, and this is due to broad distributions of the relaxation parameters. In fact, the overlapping of processes with different temperatures produces a step in the modulus whose overall amplitude is the sum of the amplitudes of the single components, whereas the height of the resulting peak of the imaginary part is smaller than the sum of the amplitudes of its components. Therefore, better estimates of the total relaxation intensities are deduced from the amplitudes of the steps in the modulus, $\frac{1}{2} \Delta Y/\Delta Y_{0.045}$ for L and 0.08 for T in Fig. 5. These values are unusually high, especially if one considers that they are found in a stoichiometric material, and apparently do not occur in concomitance with structural phase transformations. In order to better appreciate this point, one should consider that a relative softening of 0.08 means that, on application of a tensile stress, the sample undergoes an additional extension of 8% due to the relaxation of the “defects.” For this reason, the relaxations L and T can only be accounted for by intrinsic mechanisms, which must be connected with the instability of the La\textsubscript{2}CuO\textsubscript{4} lattice against certain types of deformations, like the tilting of the CuO\textsubscript{6} octahedra, and the large vibration amplitudes of the O atoms observed by neutron diffraction.\textsuperscript{46}

Figure 6 presents the whole relaxation spectrum of sample 2, from 1.4 to 700 K both in the O doped and undoped conditions. The HTT to LTO transition occurs at 530 K in the undoped state, as expected, and at 500 K after the maximum oxygenation. According to the $T, \delta$ phase diagram of Zhou \textit{et al.},\textsuperscript{4} a transition temperature $T_{d1} = 500$ K corresponds to $\delta = 0.018$. The dissipation maximum just below

\[ \begin{align*}
\text{FIG. 5. Elastic energy-loss coefficient, } Q^{-1}, \text{ and relative change of the Young’s modulus of La}_2\text{CuO}_4 \text{ (sample 1, 500 Hz and 6.2 kHz) and } \text{Nd}_2\text{CuO}_4 \text{ (840 Hz).}
\end{align*} \]

\[ \begin{align*}
\text{FIG. 6. Complete elastic susceptibilities of undoped and O-doped (}\delta = 0.018\text{) } \text{La}_2\text{CuO}_{4+\delta}. \text{ The temperatures of the HTT-LTO transformations and the main relaxation processes are indicated.}
\end{align*} \]
C. Relaxation T and the thermally activated flipping of CuO$_6$ octahedra

The main component of peak T in Fig. 3 has been fitted with $\tau_0 = 1.4 \times 10^{-12}$ s, $E/k = 2800$ K, $\alpha = 0.46$. It is considerably broader than pure Debye relaxation, and the attempt frequency is somewhat slower than that typical of atomic hopping. All these features are consistent with a coordinated motion of groups of atoms, like the proposed flipping of single CuO$_6$ octahedra or of a restricted number of them between configurations with different tilts and distortions. Since each octahedron shares the corners with the other four octahedra in the plane, the marked departure from pure Debye relaxation is a consequence of the static and dynamic interaction among these structural units.

However, the hypothesis that all the octahedra in the LTO phase may flip by thermal activation seems to be in contrast with the observation of static LTO domains with a long-range order. Actually, from a detailed neutron-diffraction study$^6$ on single crystals of La$_2$CuO$_4$ and La$_{1.87}$Sr$_{0.13}$CuO$_4$ it results that the dynamic displacements of the O atoms in the directions of the tilts are even larger than the static ones at least down to 180 K, and remain comparable to the static displacements down to the lowest temperatures. This has been interpreted as a sign that the O atoms (particularly the apical ones) can switch configuration by thermal activation.$^5$ The possibility that the relaxation processes L and/or T are directly related to the large dynamic displacements of O found by neutron diffraction is very attractive, and would also explain their extremely high intensities. Still, some care must be taken in directly identifying the dynamic displacements observed by neutron diffraction with the relaxation processes L and T, because the firsts are higher in the Sr-doped sample,$^6$ while the intensity of the latter decreases with O doping. A direct comparison between the doping dependence of the Debye-Waller displacements and of the relaxation processes L and T cannot be made yet, due to the lack of complete spectra of the imaginary part of the elastic constants. More data are available for the material with La partially substituted by Sr or Ba, where the new types of tilts are better stabilized, and the Pccn and LTT domains have actually been observed. In La$_{1.4}$Ba$_{0.6}$CuO$_4$ a step in the sound velocity was found at $T_{d1} = 50$ K (Ref. 34), when the LTO phase transforms into LTT, and in La$_{1.475}$Nd$_{0.5}$Sr$_{0.125}$CuO$_4$ two distinct dips in $\varepsilon_{66}$ were found at the transformations from LTO to Pccn and then to LTT.$^{35}$ In La$_{2-x}$Sr$_x$CuO$_4$, rises in the acoustic absorption and elastic constants have been frequently observed below 10–30 K, and attributed to the appearance of a new phase,$^{34,50,51}$ to the occurrence of a glassy state$^{31}$ or to other causes.$^{33,47}$ In some cases no acoustic anomaly was observed below 60 K.$^{52,53}$

Nohara et al.$^{54}$ also studied the interplay between lattice instability and emergence of superconductivity. They found that $\varepsilon_{66}$ [$ [(c_{11} - c_{12})/2$ in their HTT representation] starts softening below 50 K in La$_{1.86}$Sr$_{0.14}$CuO$_4$, and hardens again at the appearance of superconductivity; they could shift the onset of superconductivity and therefore extend the region of softening by applying a magnetic field. These observations have been interpreted as evidence of a structural instability of the LTO phase which is suppressed by superconductivity; the authors also noted that the in-plane shear strain coupled to the softening is responsible for a differentiation of the otherwise equivalent O atoms in the CuO$_2$ plane of the LTO phase, as for the LTT and Pccn phases.

The hardening found by us below 20 K has a nature different from that observed by Nohara et al.; in fact, it is maximum in the undoped sample, where superconductivity never sets in, and even in the highly doped state, with $T_c = 34$ K, there are no clear signs of the onset of superconductivity in the modulus. In addition, the variations of $\varepsilon_{66}$ in La$_{1.86}$Sr$_{0.14}$CuO$_4$ were below 0.3%, while the hardening in our undoped sample is up to 10%. This is shown in Fig. 7, presenting the susceptibility curves below 40 K for sample 2 from the completely outgassed state to the maximum O content that we can achieve; the decrease and shift to lower maximum at much higher temperature, slightly below the HTT-LTO transition at 500–530 K (Fig. 6).
temperature with O doping will be discussed in the next section. The inset also shows the linear correlation between the peak in the absorption and the negative step in the modulus. The energy scale of the processes, due to lattice disorder, causing a broadening of the relaxation due to the distortions of the different phases of the 214 compound, and is the most influential in determining the temperature dependence much slower than the Arrhenius law ($\tau^{-1} \propto T^{-7}$ for two-phonon processes and $\tau^{-1} \propto T$ for one-phonon processes). Then the shape of the $Q^{-1}(T)$ curve is also defined by the thermodynamic factor:

$$f_{1f_2} = \frac{1}{T} \frac{c}{\cosh^2(A/2kT)},$$

where $A$ is the energy asymmetry between the two possible defect states, due to the nonequivalence of the configurations and to internal strains. The energy scale of the processes involving tunneling is comparable with the energy shifts $A$ due to lattice disorder, causing a broadening of the relaxation function which further masks the temperature shift of the Debye function.

Support to the hypothesis that process L is of the relaxation type comes from the comparison of our measurements with those of Migliori et al., who reported the elastic constants of a La$_2$CuO$_4$ single crystal between 25 and 55 K. In Fig. 8 we put our data for stoichiometric La$_2$CuO$_4$, taken at 0.5 and 6.3 kHz, together with the temperature dependence of the $c_{66}$ constant around 1 MHz,

![FIG. 8. Comparison between the measurements on our undoped ceramic La$_2$CuO$_4$ and the temperature dependence of $c_{66}$ determined by Migliori et al. (Ref. 48) on a single crystal.](Image 60x426 to 289x733)

FIG. 8. Comparison between the measurements on our undoped ceramic La$_2$CuO$_4$ and the temperature dependence of $c_{66}$ determined by Migliori et al. (Ref. 48) on a single crystal.

The extremely broad shape of the $Q^{-1}(T)$ curves below 40 K and the fact that their shift to higher temperature at higher frequency is less marked than for the other processes, is not unusual at such low temperature, where the relaxation rate $\tau^{-1}$ is generally governed by quantum effects. Indeed, the broad shape of peak L cannot be reproduced with $\tau$ obeying the Arrhenius law for overbarrier hopping. We attempted to fit the $Q^{-1}(T)$ curves below 40 K by assuming hopping over two types of barriers, both with a distribution function whose width is up to half of its mean value, but also in this case the resulting curves consist of two well defined separate peaks, instead of the broad experimental curves. The only way to fit peak L by assuming overbarrier hopping with an attempt frequency of the order of the local atomic vibrations ($\tau_0 \sim 10^{-14}$ s) is to construct an ad hoc distribution of barriers $P(E)$ which is maximum around $E \sim 100$ K and has asymptotic behavior $E^m$ and $E^{-n}$ at low and high values of $E$, respectively, (the high-temperature side of the peak requires $n \sim 1.5$). A more physical picture is that the dynamics of relaxation L is dominated by tunneling. In fact, at progressively lower temperature the major contribution to $\tau^{-1}$ in a nonmetallic material comes from defect transitions involving two phonons and finally one phonon, which produce a temperature dependence much slower than the Arrhenius law.
The same authors attributed the minimum of $Q^{-1}$ to a relaxation process possibly connected with highly anharmonic phonon modes or domain-wall motion.

E. Influence of doping

Figure 9 presents the evolution of the peaks L and T with increasing O doping, $0 < \delta \leq 0.02$. Both peaks have a similar behavior under the introduction of small quantities of excess oxygen: they are depressed and shift to lower temperature, but peak T is much more effectively inhibited than peak L.

Also the partial substitution of La with Sr or Ba seems to inhibit these processes, but the data available in the literature are somewhat contradictory and insufficient to determine this issue. In addition, a direct comparison between low-frequency and ultrasound experiments is sometimes difficult, for example, due to the scattering of the sound waves from the domain walls in the latter type of experiments.

We try now to interpret this behavior of peaks L and T in terms of lattice disorder and electronic effects connected with the doping of charge carriers. It has been proposed that there is a mismatch between the bond lengths in the LaO planes and those in the CuO$_2$ planes, so that the CuO$_2$ planes are subjected to a compressive stress and become corrugated. This is considered as a cause of the tilting of the octahedra and therefore it has been identified as the driving force for the transition from HTT to LTO, although the observation of finite tilts also in the HTT phase indicates that the situation is more complex. Such a mismatch is described by the tolerance factor $t = [La-O]/[\sqrt{2} Cu-O] < 1$, which would be 1 in the case of stress-free flat planes. The introduction of interstitial O increases $t$ for two reasons: (i) the interstitial atoms occupy tetrahedral sites between two LaO planes, which therefore are expanded; (ii) the excess O$^2-$ removes charge from the Cu-O antibonds, so reducing the Cu-O bond lengths. High concentrations of excess O may even make $t$ exceed 1 (Ref. 24) and similar effects are produced by the substitution of La$^{3+}$ with the larger ion Sr$^{2+}$ (Ref. 59). Let us now consider the double-well potential for the local atomic configurations which produce the anelastic relaxation $T$, whose separation between the minima represents the tilt amplitude. If these double potentials are partly due to the fact that $r < 1$, then a relief of the stresses between LaO and CuO$_2$ planes should result in a decrease of both the distance between the potential minima and of the barrier, i.e., in a decrease of the activation energy; a reduction of the average tilt angle of the octahedra with doping is actually observed. The reduction of the activation energy implies a faster rate for the hopping between the possible configurations and therefore the maximum of the anelastic relaxation intensity, which occurs around $\omega \tau = 1$, is shifted to lower temperature. The same argument holds for peak L, even if the relaxation dynamics is not describable by the classical rate theory; also when tunneling dominates, a reduction of the barrier results in faster rates and therefore in a shift to lower temperature of the corresponding relaxation process. We conclude that the introduction of holes in the CuO$_2$ planes reduces the barrier between the off-center configurations and therefore accelerates the relaxation dynamics.

On the other hand, the lattice disorder, both due to interstitial O and substitutional Sr and Ba, makes the double potentials more asymmetric. This results in a suppression of the relaxation when the induced asymmetry is higher than $kT$, since the configuration with the highest energy is no more populated and the factor $f_1 f_2$ in Eqs. (1) and (2) goes to zero as exp($-A/kT$). The fact that peak T is more markedly depressed than peak L is consistent with the hypothesis of a coordinated motion of entire CuO$_6$ octahedra; since the octahedra are correlated with each other, it is possible that the pinning due to a single interstitial oxygen propagates to several structural units, causing complete blocking already at low defect concentrations. Instead, the correlation is certainly lower for single O atoms making transitions between tunneling states. The measurement with the highest content of excess O seems to contradict the above picture, because peak L in curve 6 of Fig. 9 is lower in intensity than curve 5, but shifted to higher rather than to lower temperature. This can be explained by the occurrence of the phase separation into O-rich and O-poor phases, with the O-rich phase not contributing to the relaxation process. In addition, it is possible that not all excess oxygen goes in the lattice as O$^2-$ ions, but partly forms covalent bonds, for example O$^2-$ pairs, which contribute less to the charge doping. Then, the unexpected shift to higher temperature of peak L at the highest O contents may be explained by the formation of covalent pairs which contribute only partially to doping, and therefore to the acceleration of the relaxation dynamics, but are effective in creating disorder and therefore in suppressing the intensity.

The conclusion that O atoms in La$_2$CuO$_4+\delta$ may tunnel and that their dynamics becomes faster on increasing doping poses the question whether this tunneling dynamics may play a role in the superconducting mechanism, since many models...
have appeared of strong electron-lattice coupling mediated by tunneling states.\textsuperscript{60,61} Experimental evidences of double-well potentials along the $z$ axis for the apical O atoms in 123 and Ti-based oxide superconductors have been provided mainly by extended x-ray-absorption fine-structure measurements,\textsuperscript{62} but are still controversial.\textsuperscript{63,64} In the present case there is evidence of an extraordinarily intense relaxation involving tunnel states, but it also seems that this relaxation mechanism loses importance at the doping levels at which the material becomes metallic and superconducting, at least for doping with interstitial O. Additional acoustic experiments may clarify this point.

\textbf{IV. CONCLUSIONS}

The present measurements show the existence of four anelastic relaxation processes in La$_2$CuO$_{4+\delta}$ which depend on the O stoichiometry and may reach very high intensities. Two of them are clearly connected with the presence of excess O, and from the analysis of the elastic energy loss peak due to hopping of single interstitial O atoms it has been possible to accurately determine their mobility.

It is also shown that, after eliminating the excess O, a relevant fraction of the lattice in the LTO phase presents relaxational dynamics on three different time scales, which is a manifestation of the intrinsic instability of the La$_2$CuO$_4$ lattice against certain types of deformations, like the tilting of the CuO$_6$ octahedra. This is in sharp contrast with Nd$_2$CuO$_4$, which has a stable $T'$ structure without CuO$_6$ octahedra and a flat dynamic elastic susceptibility. The only transformation which is known to occur in undoped La$_2$CuO$_4$ is the HTT-LTO one, and the movement of the twin walls in the LTO domains cannot be responsible for three relaxation processes with widely different characteristic energies and temperatures (450, 150, and 5 K). The slowest relaxation, observed below the HTT-LTO transformation temperature, is attributed to the motion of the twin boundaries. The intermediate process, with relaxation times of the order of 1 ms around 150 K, is due to hopping over a barrier of $\sim 0.23$ eV; it is identified with thermally activated flipping of unstable CuO$_6$ octahedra, possibly in the domain boundaries, and is easily blocked by the lattice disorder introduced by excess oxygen. The faster relaxation motion, which is less effectively blocked by the interstitial excess O, has characteristic times of the order of 1 ms around 5–10 K and is dominated by quantum tunneling. It is proposed that the double-well potentials for the low-temperature relaxation are due to the tendency of the LTO phase to nucleate LTT domains. In stoichiometric La$_2$CuO$_4$ however, these domains cannot be stabilized, as in the doped lattice, by the mismatch between the sizes of the La ions and those which partially substitute it, or by the commensurate charge-density wave effect when doping is $\sim 1/8$. Therefore, possibly near the twin walls, the O sublattice can locally assume an LTT-like pattern, which however remains localized and short lived.

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